2020

MATHEMATICS — HONOURS

Eighth Paper

(Module - XV)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols used in the question paper carry usual meaning.

Group - A

(Marks : 25)

Section - I

Answer *any two* questions.

1. (a) Prove that $E \equiv 1 + \dfrac{\delta^2}{2} + \delta\sqrt{1 + \dfrac{\delta^2}{4}}$.

   (b) The absolute error in the common logarithm of an approximate number is less than half the relative error of the number. — Justify or rectify it. 6+4

2. (a) Find the second-order divided difference of $\sin x$ for the arguments $\dfrac{\pi}{6}, \dfrac{\pi}{4}, \dfrac{\pi}{2}$.

   (b) Define confluent divided difference $f(x_0, x_0)$ of the function $f(x)$. Show that the first-order divided difference $f(x_0, x_1)$ of the function $f(x)$ is independent of its arguments x_0, x_1 if and only if $f(x)$ is a linear polynomial of x. 4+(2+4)

3. (a) Write down Lagrange's linear interpolation polynomial for interpolating $f(x)$ at the distinct points x_0 and x_1.

   (b) Establish a polynomial interpolation formula with at least three terms (without error term) for $h(x)$ for the given set of $n + 1$ equally spaced nodes $x_0, x_1, x_2, ..., x_n$ ($x_0 < x_1 < x_2 < ...... < x_n$) to be used for a point near to the node x_n. 2+8

4. Obtain the composite form of Simpson's $\dfrac{1}{3}$ rule. Why composite form is used? State with reason whether Simpson's $\dfrac{1}{3}$ rule is an open type or a closed type formula. 6+2+2

Please Turn Over

5. (a) Explain the principle of numerical differentiation.

 (b) If the value of the integral $\int_0^1 \left(x^{19} - cx^9 \right) dx$ is exact when it is computed by simple trapezoidal rule,

 find the value of c. Does this contradict that trapezoidal rule has the degree of precession one? Justify. 4+4+2

Section - II

Answer *any one* question.

6. Describe regula-falsi method to determine approximately a real simple root of an equation $f(x) = 0$. Why does the method call "linear interpolation method"? 3+2

7. Describe Gauss's elimination method with partial pivoting to find the approximate roots of a simultaneous system of 4 linear equations in 4 unknowns. How does Gauss's elimination method with partial pivoting decide whether the system has unique solution or not? Is there any role of partial pivoting to guard round off error during the elimination process? Justify. 3+1+1

8. (a) Explain the method of fixed point iteration for approximating a simple real root α of an equation of the form $x = \varphi(x)$, where $\varphi(x)$ and $\varphi'(x)$ are continuous on an interval about α.

 (b) Derive a sufficient condition of convergence of the above method.

 (c) Find also the order of convergence of the above method if $\varphi'(\alpha) \neq 0$. 2+2+1

9. Given $\dfrac{dy}{dx} = 1 - \dfrac{y^2}{x}$, where $y(2) = 2$. Compute $y(2.2)$ by Euler's method, correct to four decimal places, taking $h = 0.05$. 5

Group - B

(Marks : 25)

Section - I

Answer *any one* question.

10. (a) What do you understand by a source and an object program? What is the role of compiler in connecting these programs? 2+2+2

 (b) Find the value of x by 2's complement method : 2×3
$$(x)_8 + (10101)_2 = (2B)_{16}$$

 (c) If $(101011)_2 = (53)_r$, then find the value of r. 2×2

 (d) What is the difference between if-else and switch case in C? 2×2

Or,

What is the difference between Do-loop and Implied Do-loop in FORTRAN? 2×2

11. (a) Write an algorithm to compute the sum of the first n terms of the divergent series :

$$1+\frac{1}{4}+3+1+\frac{1}{4}+3+1+\frac{1}{4}+3+\ldots\ldots$$

(b) Write a program in FORTRAN 77/90 or C to read two square matrices A and B of order 3. Hence write a program to find $A + B$.   10+(6+4)

12. (a) Draw a flowchart for the evaluation of the polynomial $f(x)=ax^3+bx^2+cx+d$ for $x = 0$ to 1 increasing in steps 0.1 and for given value of a, b, c, and d.

(b) Write a C or FORTRAN program for finding k^{th} smallest number of a given list of n distinct real numbers.   10+10

13. (a) Write a C or FORTRAN program to evaluate $\displaystyle\int_{\frac{1}{7}}^{\frac{1}{2}}\left(\sin^{-1}x+\tan^{-1}x\right)dx$ by Simpson's $\dfrac{1}{3}$ rule with subintervals 12, 24.

(b) Write an efficient program in C or FORTRAN 77/90 to solve the following differential equation by 4th order Runge-Kutta method correct up to 6 decimal places : $\dfrac{dy}{dx}=\dfrac{x+y^2}{x^2+y^3}$ for $x = 0{\cdot}1(0{\cdot}1)1{\cdot}0$, given $y = 1{\cdot}0$ for $x = 0{\cdot}0$.   10+10

Section - II

Answer **any one** question.

14. (a) Prove that the set of all positive divisors of 70 forms a Boolean algebra $(S, +, \bullet, ')$ where for a, $b \in S$ $a + b$ = the l.c.m of a, b; $a.b$ = the g.c.d of a, b; $a' = 70/a$.

(b) Let $B(+, \bullet, ')$ be a Boolean algebra and a, b, $c \in B$. Prove, not using truth table, that $ab + a'b' + bc = ab + a'b' + a'c$.   3+2

15. (a) Express $E(x, y, z) = (x'y + z)'\bullet(x + y'z)$ in its Conjunctive Normal Form.

(b) Find a simpler equivalent Switching Circuit for that shown in the following figure :   2+3